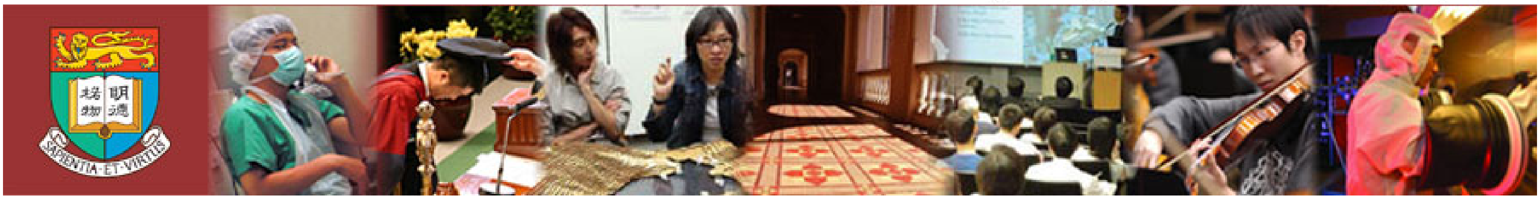

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Security-Constrained Dispatch with Controllable Loads for Integrating Stochastic Wind Energy

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***Abstract*—This paper presents a bi-level consumer-utility optimization model to schedule an energy consumption pattern of controllable loads in the face of a time varying price function depending on system conditions and market operations. The controllable loads are classified into three types based on their natures and operating characteristics for an upper-level consumer's problem. To formulate the stochastic wind generators, a security-constrained optimal power flow (SCOPF) model is proposed for a lower-level utility's problem to consider various wind power scenarios. We then convert the bi-level model into a single-level of mathematical program with equilibrium constraints' (MPECs) problem to obtain the optimal load scheduling results. Finally, a simple case study is conducted to demonstrate the feasibility of the method.**

***Index Terms*—Controllable loads, security-constrained optimal power flow (SCOPF), energy pricing, bi-level model, mathematical program with equilibrium constraints (MPECs).**

I. NOMENCLATURE

A. Indices

t	time index
i	generator index
j	*type I load index*
k	*type II load index*
l	*type III controllable load index*

B. Set

T	set of indices of time
I	set of indices of generators
J	set of indices of type I loads
K	set of indices of type II loads
L	set of indices of type III loads
Ω_j	set of indices of allowed operating hours of type I loads
Ω_k	set of indices of allowed operating hours of type II loads
Ω_l	set of indices of allowed operating hours of type III loads

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C. Constant

P_j^I	Rated power output of type I load j
H_j^I	Total operating period of type I load j
P_k^{II}	Rated power output of type II load k
p_k^{II}	Minimum operating period of type II load k
q_k^{II}	Time constraint of type II load k
t_{max}	Maximum time index
$P_l^{III,min}$	Minimum power output of type III load l
$P_l^{III,max}$	Maximum power output of type III load l
$E_l^{III,min}$	Minimum energy consumption of type III load l
$C_{i,t}$	Energy offer cost of generator i at period t
$C_{i,t}^U$	Up-reserve offer cost of generator i at period t
$C_{i,t}^D$	Down-reserve offer cost of generator i at period t
π_s	Probability of wind power scenario s
$P_i^{g,max}$	Maximum power output of generator i
$P_i^{g,min}$	Minimum power output of generator i
P_t^{BL}	Baseload power at period t
$R_i^{U,max}$	Maximum up-reserve provided by generator i
$R_i^{D,max}$	Maximum down-reserve provided by generator i

D. Functions

L	Lagrangian function of the lower-level utility's problem

E. Continuous variables

$P_{j,t}^I$	Power output of type I load j at period t
$P_{k,t}^{II}$	Power output of type II load k at period t
$P_{l,t}^{III}$	Power output of type III load l at period t
μ_t	Marginal energy price at period t
$P_{i,t}^g$	Power output of generator i at period t
$R_{i,t}^U$	Up-reserve scheduled by generator i at period t
$R_{i,t}^D$	Down-reserve scheduled by generator i at period t
$r_{i,t,s}^U$	Up-reserve deployed by generator i at period t and scenario s
$r_{i,t,s}^D$	Down-reserve deployed by generator i at period t and scenario s
$\lambda_{t,0}$	Lagrange multiplier associated with the power balance equation at period t in market scheduling state
$\lambda_{t,s}$	Lagrange multiplier associated with the power balance equation at period t and scenario s

$\gamma_{i,t,0}^{max}$ Lagrange multiplier associated with the upper bound for the power output of generator i at period t in market scheduling state

$\gamma_{i,t,0}^{min}$ Lagrange multiplier associated with the lower bound for the power output of generator i at period t in market scheduling state

$\gamma_{i,t,s}^{max}$ Lagrange multiplier associated with the upper bound for the power output of generator i at period t and scenario s

$\gamma_{i,t,s}^{min}$ Lagrange multiplier associated with the lower bound for the power output of generator i at period t and scenario s

$\alpha_{i,t,s}^{max}$ Lagrange multiplier associated with the upper bound for the deployed up-reserve of generator i at period t and scenario s

$\alpha_{i,t,s}^{min}$ Lagrange multiplier associated with the lower bound for the deployed up-reserve of generator i at period t and scenario s

$\beta_{i,t,s}^{max}$ Lagrange multiplier associated with the upper bound for the deployed down-reserve of generator i at period t and scenario s

$\beta_{i,t,s}^{min}$ Lagrange multiplier associated with the lower bound for the deployed down-reserve of generator i at period t and scenario s

$\delta_{i,t}^{max}$ Lagrange multiplier associated with the upper bound for the scheduled up-reserve of generator i at period t

$\upsilon_{i,t}^{max}$ Lagrange multiplier associated with the upper bound for the scheduled down-reserve of generator i at period t

F. Binary variables

$x_{j,t}^{I}$ 0/1 variable that is equal to 1 if type I load j is on and otherwise is equal to 0 at period t

$x_{k,t}^{II}$ 0/1 variable that is equal to 1 if type II load k is on and otherwise is equal to 0 at period t

$x_{l,t}^{III}$ 0/1 variable that is equal to 1 if type III load l is on and otherwise is equal to 0 at period t

G. Random variables

$P_{t}^{w,sche}$ Scheduled wind power generation at period t

$P_{t,s}^{w}$ Wind power generation in period t and scenario s

II. Introduction

TRADITIONALLY, power balance is achieved by regulating a controllable generation to meet an inelastic demand. With increasing concern on climate change, renewable resources (primarily in wind energy) are more widely used in the electricity sector to reduce carbon emissions. Since renewables are intermittent, their power outputs are stochastic and difficult to predict accurately [1]. This introduces an immense challenge to facilitate power balance from only the generation sector. As such, demand response (DR) is also an effective method for utilities to help maintaining system reliability under contingencies [2]-[4].

Sophisticated DR algorithms were proposed in a smart grid environment to schedule the consumption patterns of end-users [5]-[8]. Du and Lu [5] proposed an appliance commitment to find an optimal schedule of thermostatically controlled appliances based on price and consumption forecasts. Mohsenian-Rad and Leon-Garcia [6] introduced an optimization framework to schedule the operation and energy consumption of residential appliances by minimizing the household's electricity payment. Pedrasa *et al* [7] described a distributed energy resources scheduling algorithm to maximize the end user's revenue in a smart home case study. Li *et al* [8] presented a DR approach based on utility maximization to operate different appliances including PHEVs and batteries. Although "smart" DR algorithms have been extensively proposed, those approaches generally neglected the relationship between demand sides and system conditions and formulated price signals as a predefined function which cannot reflect the dynamic nature of market operations.

To formulate the production variability from renewable resources and the dynamic pricing for DR, we propose a bi-level consumer-utility optimization model with a marginal electricity market price to schedule an optimal energy consumption pattern of controllable loads in the systems with stochastic wind generation.

The upper level is a consumer's problem which is assumed to be a leader. In the consumer's problem, we classify the controllable loads into three types as shown in our previous work [9]. The objective of the consumer's problem is to minimize the electricity payment by scheduling its daily operation schedule based on the time-varying price signals.

On the other hand, the lower level is a utility's problem which use a multi-period security-constrained optimal power flow (SCOPF) model [10]-[13]. The SCOPF model is a scenario-based approach to formulate the stochastic wind power production into different discrete scenarios and enforces those scenarios in the power balance and power production constraints [12].

Traditional approach for some electricity markets is to schedule energy and various reserve services in a sequential market-clearing procedure [14]-[16]. However, the interrelationship between energy and reserves services is so strong in the smart grid due to the highly volatile fluctuation of wind power outputs. Another approach which is called a simultaneously market-clearing procedure [17]-[19] is therefore used in this paper. The simultaneous market-clearing procedure mainly co-optimizes energy and reserve services in a coordinated manner. The SCOPF model is a typical one with this procedure and it simultaneously considers various market products such as energy, reserve capacity and actual reserve deployment in two states, namely market scheduling stare and actual operation state [10], [13]. With the SCOPF model and the market clearing procedure, the utility's problem aims to determine the optimal level of energy and reserves with the consideration of controllable loads given by the consumer's problem.

Since this paper is not focus on pricing scheme of the SCOPF model, we simply use the marginal pricing scheme presented in [10] for the utility's problem to make the proper economic signals to the consumer. The pricing scheme of the

SCOPF model mainly relies on the marginal approach [20] that uses Lagrangian multipliers on the power balance equations of both market scheduling and actual operation states to represent energy prices [13]. The marginal energy prices are derived from the Karush-Kuhn-Tucker (KKT) optimality conditions which are more realistic to reflect the opportunity costs of energy and reserve services in the high penetration of wind energy.

Since the utility's problem is continuous and convex, we can derive the KKT conditions of the lower-level utility's problem as the equilibrium constraint in the upper-level consumer's problem. Therefore, the bi-level formulation is converted into a form of mathematical program with equilibrium constraints (MPECs) [21] which can be solved directly in GAMS [22].

The bi-level formulation is commonly known as a "minimax" programming which optimizes two conflicting objectives with characterizing uncertainties into the different discrete scenarios [23]. On the other hand, another common stochastic programming such as chance-constrained approach uses probabilistic distributions to handle the stochastic variables, and therefore requires a Monte Carlo simulation to generate a large number of scenarios [24].

Comparing with the chance-constrained approach, the "minimax" programming has the advantage of efficient computation because it can be solved deterministically without the need of Monte Carlo simulations. However, this "minimax" method may introduce an overly conservative procurement of reserve capacity in our problem since "low-wind" scenarios are more frequently occurred in practice. In contrast, the chance-constrained approach may accommodate a more versatile treatment of uncertainty in the constraints [25] but at meanwhile, a heuristic and complex solution algorithm is required to solve this approach.

This paper is organized as follow. Section III formulates the proposed bi-level model. Section IV provides a solution approach of the problem. Section V gives a numerical example of the model and finally, some conclusions are drawn in Section VI.

III. Bi-level Consumer-Utility Smart Dispatch Model

In this section, we provide a mathematical formulation of the bi-level consumer-utility model. The upper level is the consumer's problem which aims to use controllable loads to minimize the payment. The utility's problem is the SCOPF model for market clearing at the lower level.

A. Assumptions

To focus on evaluating the impact of renewable sources on system operation, the only stochastic part in this paper is wind power output. In order words, the set of pre-specified actual operation states in the SCOPF model is various wind power scenarios. The market structure for the proposed model is assumed in a day-ahead market with the marginal pricing scheme.

B. Upper-level Consumer's Problem

The consumer obtains the hourly price signal from the utility and manages the energy consumption of three types of controllable loads in order to minimize the total electricity payment. The controllable loads are divided into three types according to their natures and operational characteristics. The mathematical formulation of the consumer's problem is shown as follows:

$$min \sum_{t \in T} \mu_t(D_t) \left(\sum_{j \in J} P_{j,t}^I + \sum_{k \in K} P_{k,t}^{II} + \sum_{l \in L} P_{l,t}^{III} \right) \tag{1}$$

subject to

1) Constraints of type I load:

$$P_{j,t}^I = x_{j,t}^I P_j^I, \forall j \in J \tag{2}$$

$$\sum_{t \in T} x_{j,t}^I = H_j^I, \forall j \in J \tag{3}$$

2) Constraints of type II load:

$$P_{k,t}^{II} = x_{k,t}^{II} P_k^{II}, \forall k \in K \tag{4}$$

$$x_{k,t}^{II} + x_{k,t+1}^{II} + \cdots + x_{k,t+q_k^{II}}^{II} \ge p_k^{II}, \forall k \in K, \forall t = 1,2,\ldots,t_{max} - q_k^{II} \tag{5}$$

3) Constraints of type III load:

$$x_{l,t}^{III} P_l^{III,min} \le P_{l,t}^{III} \le x_{l,t}^{II} P_l^{III,max}, \forall l \in L \tag{6}$$

$$\left(\sum_{t \in T} P_{l,t}^{III} \right) \Delta t \ge E_l^{III,min} \tag{7}$$

4) Operating period constraints:

$$\begin{cases} x_{j,t}^I \in \{0,1\} | t \in \Omega_j \\ x_{j,t}^I = 0 | t \notin \Omega_j \end{cases}, \forall j \in J \tag{8}$$

$$\begin{cases} x_{k,t}^{II} \in \{0,1\} | t \in \Omega_k \\ x_{k,t}^{II} = 0 | t \notin \Omega_k \end{cases}, \forall k \in K \tag{9}$$

$$\begin{cases} x_{l,t}^{III} \in \{0,1\} | t \in \Omega_l \\ x_{l,t}^{III} = 0 | t \notin \Omega_l \end{cases}, \forall l \in L \tag{10}$$

The objective function of the consumer is to minimize the total electricity payment as shown in (1). We use a time-varying price function $\mu_t(D_t)$ which depends on the aggregate consumption level D_t to model a dynamic electricity price. The detailed derivation of this price function in terms of system conditions will be given in Section III-D.

Type I load is defined as a load which consumes fixed amount of total energy consumption per day. Some typical examples of type I loads are water pump, dish washer, washing machine and electric vehicle charging. Equation (2) imposes that type I load is a single-mode appliance. It can be on and off arbitrary within the day but the fixed energy has to be applied before the last allowed operating hour as shown in (3).

The operation of a type II load is similar to the type I load, but with time constraints in continuous time periods. For example, a refrigerator can be temporally switched off when the electricity price is high. However, it cannot be turned off more than few hours (e.g. 2 hours) in order to keep the food fresh. Some other typical appliances are freezer, water heater and air conditioner. The type II load is formulated as a single-mode appliance as shown in (4). Equation (5) ensures that some energy has to be given to the type II load constrained with the time limit q_k^{II}.

A type III load mainly includes the cooking and entertainment appliances which consume flexible amount of energy over the entire day as shown in (6). To ensure maintaining certain comfortability to customers, a minimum energy is required to be given within the schedule period as shown in (7).

Equations (8)-(10) impose operating period constraints of different types of controllable loads. To increase the control flexibility from the demand sector, the consumer is required to input the set of allowed operating hours of controllable loads. For the hours that excludes in the set, the corresponding decision variables are forced to 0 (i.e. off).

C. Lower-level Utility's Problem

The utility's problem is modeled as the multi-period SCOPF model. The SCOPF model consists of two states, namely market scheduling and actual operation states. The energy and reserve capacities are simultaneously scheduled in both states. The objective of the utility's problem is to minimize the expected cost (EC) in a day-ahead market as shown in (11):

$$minEC = min\left\{\sum_{t\in T}\sum_{i\in I} C_{i,t}P_{i,t}^{g} + \sum_{t\in T}\sum_{i\in I}\left(C_{i,t}^{U}R_{i,t}^{U} + C_{i,t}^{D}R_{i,t}^{D}\right) + \sum_{s\in S}\pi_{S}\sum_{t\in T}\sum_{i\in I} C_{i,t}\left(r_{i,t,s}^{U} - r_{i,t,s}^{D}\right)\right\} \quad (11)$$

The objective function is subject to following constraints:

1) Power balance constraints:

a) Market scheduling states

$$\sum_{i\in I} P_{i,t}^{g} + P_{t}^{w,sche} = \sum_{j\in J} P_{j,t}^{I} + \sum_{k\in K} P_{k,t}^{II} + \sum_{l\in L} P_{l,t}^{III} + P_{t}^{BL} : (\lambda_{t,0}), \forall t \in T \quad (12)$$

b) Actual operation states:

$$\sum_{i\in I} P_{i,t}^{g} + \sum_{i\in I} r_{i,t,s}^{U} - \sum_{i\in I} r_{i,t,s}^{D} + P_{t,s}^{w} = \sum_{j\in J} P_{j,t}^{I} + \sum_{k\in K} P_{k,t}^{II} + \sum_{l\in L} P_{l,t}^{III} + P_{t}^{BL} : (\lambda_{t,s}), \forall t \in T, \forall s \in S \quad (13)$$

2) Power production constraints:

a) Market scheduling states

$$P_{i}^{g,min} \le P_{i,t}^{g} \le P_{i}^{g,max} : (\gamma_{i,t,0}^{min}, \gamma_{i,t,0}^{max}), \forall i \in I, \forall t \in T \quad (14)$$

b) Actual operation states:

$$P_{i,t}^{g} + r_{i,t,s}^{U} \le P_{i}^{g,max} : (\gamma_{i,t,s}^{max}), \forall i \in I, \forall t \in T, \forall s \in S \quad (15)$$

$$P_{i}^{g,min} \le P_{i,t}^{g} - r_{i,t,s}^{D} : (\gamma_{i,t,s}^{min}), \forall i \in I, \forall t \in T, \forall s \in S \quad (16)$$

3) Constraints linking reserve capacities

$$0 \le r_{i,t,s}^{U} \le R_{i,t}^{U} : (\alpha_{i,t,s}^{min}, \alpha_{i,t,s}^{max}), \forall i \in I, \forall t \in T, \forall s \in S \quad (17)$$

$$0 \le r_{i,t,s}^{D} \le R_{i,t}^{D} : (\beta_{i,t,s}^{min}, \beta_{i,t,s}^{max}), \forall i \in I, \forall t \in T, \forall s \in S \quad (18)$$

$$R_{i,t}^{U} \le R_{i}^{U,max} : (\delta_{i,t}^{max}), \forall i \in I, \forall t \in T \quad (19)$$

$$R_{i,t}^{D} \le R_{i}^{D,max} : (\upsilon_{i,t}^{max}), \forall i \in I, \forall t \in T \quad (20)$$

The utility's objective function to be minimized as shown in (11) consists of following five terms:

- The first term is the energy production cost.
- The second and third terms are the offered costs of generators for scheduling up- and down-reserve. The up- and down-reserve are used to accommodate the unexpected fluctuations of wind power. The up-reserve is defined as a reserve capacity which schedules for a sudden lack of power output such as unexpected decrease of wind generators' output [10]. On the other hand, the down-reserve is scheduled to absorb a sudden increase of power output such as unexpected wind power spillage. The consideration of down-reserve can help to reduce a switching of generators.
- The last two terms are the costs of up- and down–reserve deployed by generators in the actual operation state. Since the actual operation state consists of a set of wind power scenarios, the last two terms are associated with the probability of each pre-specified wind power output. It should be noted that the deployment of down-reserve can reduce the cost as shown in the fifth term in (11).

For simplicity, network constraints are ignored and the utility's problem in this paper only focuses on power balance. The power balances in the market scheduling and actual operation states are shown in (12) and (13), respectively. The approximate wind power is first forecasted in the market scheduling state.

Equation (14) shows the power production limits in market scheduling states. Equations (15) and (16) relate the up- and down-reserve limitations to the power outputs produced in the actual operation states, respectively.

Equations (17) and (18) impose the bounds of the deployed up- and down-reserve, respectively. From (17) and (18), it is noted that the upper bounds of deployed down- and up-reserve are actually the scheduled up- and down-reserve. Finally, (19) and (20) shows the upper limits of scheduled up- and down-reserve, respectively. The Lagrange multipliers associated with equations (12)-(20) are given in corresponding parentheses. It should be noted that all the Lagrange multipliers are non-negative numbers.

D. Dynamic Pricing Scheme

The time-varying price function $\mu_t(D_t)$ plays a key role to link the consumer's problem with the utility's problem. To explicitly represent system conditions in this function, we propose to use the marginal pricing scheme to provide economic signals to the consumer. This can be achieved by deriving a relationship between $\mu_t(D_t)$ and the Lagrange multipliers of the power balance constraints in both market scheduling and actual operation states.

The Lagrange multipliers in (12) and (13) are also commonly known as Locational Marginal Prices (LMPs) [26] and they embody the energy prices which can serve the consumer as market incentives. However, the marginal pricing scheme for the SCOPF model involves a complex pricing issue which is out of scope in this paper. As such, we simply use the energy pricing schemes as derived in the Appendix of [10] to derive the time-varying price function $\mu_t(D_t)$ in terms of the LMPs and the result is given as follows:

$$\mu_t(D_t) = \lambda_{t,0} + \sum_{s \in S} \lambda_{t,s} \tag{21}$$

Equation (21) states that the marginal energy price in the SCOPF model is a marginal cost of satisfying the power balance equations in both marketing scheduling state (12) and actual operation state (13). Since reserves are to maintain system security for different possible actual operation states, $\sum_{s \in S} \lambda_{t,s}$ should be included in the marginal energy price to denote the marginal cost of security [10], [11].

To verify the physical meaning of (21), we first derive the marginal energy cost from the KKT conditions in the utility's problem as shown in (22)

$$\begin{aligned}\frac{\partial L}{\partial P_{i,t}^{g}} = C_{i,t} - \left(\lambda_{t,0} - \gamma_{i,t,0}^{max} + \gamma_{i,t,0}^{min}\right)\\ - \sum_{s \in S}\left(\lambda_{t,s} - \gamma_{i,t,s}^{max} + \gamma_{i,t,s}^{min}\right)\\ = 0, \forall i \in I, \forall t \in T\end{aligned} \tag{22}$$

Given the marginal energy price as defined in (21), we can deduce the following relationship

$$\begin{aligned}\mu_t(D_t) = C_{i,t} + \left(\gamma_{i,t,0}^{max} - \gamma_{i,t,0}^{min}\right)\\ + \sum_{s \in S}\left(\gamma_{i,t,s}^{max} - \gamma_{i,t,s}^{min}\right), \forall i \in I, \forall t \in T\end{aligned} \tag{23}$$

From (23), it can be seen that the marginal energy price is also equivalent to the sum of three terms. The first term is the energy offer cost. The second and third terms are associated with the Lagrange multipliers of generation limits in market scheduling and actual operation states, respectively. Therefore, the price given in (21) is physically consistent with (23) and it can fulfill the common requirement for the marginal pricing [20].

IV. Single-level Equivalent

A. MPEC Reformulation

To solve the proposed bi-level model, we have to derive the first order Karush-Kuhn-Tucker (KKT) optimality conditions of the lower-level utility's problem as the equilibrium constraints in the upper-level consumer's problem [21]. The bi-level model is then converted to the MPECs' problem. The derivation of the KKT conditions is applicable since the decision variables of lower-level utility's problem are continuous and therefore, the utility's problem is convex. Also, the binary variables which appear in the power balance constraints as shown in (12) and (13) can be regarded as parameters to the utility's problem.

Therefore, the original bi-level problem (1)-(20) is converted into the MPECs' problem:

$$min \sum_{t \in T} \mu_t(D_t)\left(\sum_{j \in J} P_{j,t}^{I} + \sum_{k \in K} P_{k,t}^{II} + \sum_{l \in L} P_{l,t}^{III}\right) \tag{24}$$

subject to

$$P_{j,t}^{I} = x_{j,t}^{I} P_{j}^{I}, \forall j \in J \tag{25}$$

$$\sum_{t \in T} x_{j,t}^{I} = H_j^{I}, \forall j \in J \tag{26}$$

$$P_{k,t}^{II} = x_{k,t}^{II} P_{k}^{II}, \forall k \in K \tag{27}$$

$$\begin{aligned}x_{k,t}^{II} + x_{k,t+1}^{II} + \cdots + x_{k,t+q_k^{II}}^{II} \geq p_k^{II}, \forall k \in K, \forall t\\ = 1,2, \ldots, t_{max} - q_k^{II}\end{aligned} \tag{28}$$

$$x_{l,t}^{III} P_l^{III,min} \leq P_{l,t}^{III} \leq x_{l,t}^{III} P_l^{III,max}, \forall l \in L \tag{29}$$

$$\left(\sum_{t \in T} P_{l,t}^{III}\right)\Delta t \geq E_l^{III,min} \tag{30}$$

$$\begin{cases} x_{j,t}^{I} \in \{0,1\} | t \in \Omega_j \\ x_{j,t}^{I} = 0 | t \in \Omega_j \end{cases}, \forall j \in J \tag{31}$$

$$\begin{cases} x_{k,t}^{II} \in \{0,1\} | t \in \Omega_k \\ x_{k,t}^{II} = 0 | t \in \Omega_k \end{cases}, \forall k \in K \tag{32}$$

$$\begin{cases} x_{l,t}^{III} \in \{0,1\} | t \in \Omega_l \\ x_{l,t}^{III} = 0 | t \in \Omega_l \end{cases}, \forall l \in L \tag{33}$$

$$\begin{aligned}\sum_{i \in I} P_{i,t}^{g} + P_t^{w,sche} = \sum_{j \in J} P_{j,t}^{I} + \sum_{k \in K} P_{k,t}^{II} + \sum_{l \in L} P_{l,t}^{III}\\ + P_t^{BL}, \forall t \in T\end{aligned} \tag{34}$$

$$\begin{aligned}\sum_{i \in I} P_{i,t}^{g} + \sum_{i \in I} r_{i,t,s}^{U} - \sum_{i \in I} r_{i,t,s}^{D} + P_{t,s}^{w}\\ = \sum_{j \in J} P_{j,t}^{I} + \sum_{k \in K} P_{k,t}^{II} + \sum_{l \in L} P_{l,t}^{III}\\ + P_t^{BL}, \forall t \in T, \forall s \in S\end{aligned} \tag{35}$$

$$P_i^{g,min} \leq P_{i,t}^{g} \leq P_i^{g,max}, \forall i \in I, \forall t \in T \tag{36}$$

$$P_{i,t}^{g} + r_{i,t,s}^{U} \leq P_i^{g,max}, \forall i \in I, \forall t \in T, \forall s \in S \tag{37}$$

$$P_i^{g,min} \leq P_{i,t}^{g} - r_{i,t,s}^{D}, \forall i \in I, \forall t \in T, \forall s \in S \tag{38}$$

$$0 \leq r_{i,t,s}^{U} \leq R_{i,t}^{U}, \forall i \in I, \forall t \in T, \forall s \in S \tag{39}$$

$$0 \leq r_{i,t,s}^{D} \leq R_{i,t}^{D}, \forall i \in I, \forall t \in T, \forall s \in S \tag{40}$$

$$R_{i,t}^{U} \leq R_i^{U,max}, \forall i \in I, \forall t \in T \tag{41}$$

$$R_{i,t}^{D} \leq R_i^{D,max}, \forall i \in I, \forall t \in T \tag{42}$$

$$\begin{aligned}C_{i,t} - \left(\lambda_{t,0} - \gamma_{i,t,0}^{max} + \gamma_{i,t,0}^{min}\right)\\ - \sum_{s \in S}\left(\lambda_{t,s} - \gamma_{i,t,s}^{max} + \gamma_{i,t,s}^{min}\right)\\ = 0, \forall i \in I, \forall t \in T\end{aligned} \tag{43}$$

$$C_{i,t}^{U}-\sum_{s\epsilon S}\alpha_{i,t,s}^{max}+\delta_{i,t}^{max}=0,\forall i\in I,\forall t\in T \quad (44)$$

$$C_{i,t}^{D}-\sum_{s\epsilon S}\beta_{i,t,s}^{max}+\upsilon_{i,t}^{max}=0,\forall i\in I,\forall t\in T \quad (45)$$

$$\pi_s C_{i,t}-\lambda_{t,s}+\gamma_{i,t,s}^{max}-\alpha_{i,t,s}^{min}+\alpha_{i,t,s}^{max}=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (46)$$

$$-\pi_s C_{i,t}+\lambda_{t,s}+\gamma_{i,t,s}^{min}-\beta_{i,t,s}^{min}+\beta_{i,t,s}^{max}=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (47)$$

$$\gamma_{i,t,0}^{max}\geq 0,\forall i\in I,\forall t\in T \quad (48)$$

$$\gamma_{i,t,0}^{min}\geq 0,\forall i\in I,\forall t\in T \quad (49)$$

$$\gamma_{i,t,s}^{max}\geq 0,\forall i\in I,\forall t\in T,\forall s\in S \quad (50)$$

$$\gamma_{i,t,s}^{min}\geq 0,\forall i\in I,\forall t\in T,\forall s\in S \quad (51)$$

$$\alpha_{i,t,s}^{max}\geq 0,\forall i\in I,\forall t\in T,\forall s\in S \quad (52)$$

$$\alpha_{i,t,s}^{min}\geq 0,\forall i\in I,\forall t\in T,\forall s\in S \quad (53)$$

$$\beta_{i,t,s}^{max}\geq 0,\forall i\in I,\forall t\in T,\forall s\in S \quad (54)$$

$$\beta_{i,t,s}^{min}\geq 0,\forall i\in I,\forall t\in T,\forall s\in S \quad (55)$$

$$\delta_{i,t}^{max}\geq 0,\forall i\in I,\forall t\in T \quad (56)$$

$$\upsilon_{i,t}^{max}\geq 0,\forall i\in I,\forall t\in T \quad (57)$$

$$\gamma_{i,t,0}^{max}\left(P_i^{g,max}-P_{i,t}^{g}\right)=0,\forall i\in I,\forall t\in T \quad (58)$$

$$\gamma_{i,t,0}^{min}\left(P_{i,t}^{g}-P_i^{g,min}\right)=0,\forall i\in I,\forall t\in T \quad (59)$$

$$\gamma_{i,t,s}^{max}\left(P_i^{g,max}-P_{i,t}^{g}-r_{i,t,s}^{U}\right)=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (60)$$

$$\gamma_{i,t,s}^{min}\left(P_{i,t}^{g}-P_i^{g,min}-r_{i,t,s}^{D}\right)=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (61)$$

$$\alpha_{i,t,s}^{max}\left(R_{i,t}^{U}-r_{i,t,s}^{U}\right)=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (62)$$

$$\alpha_{i,t,s}^{min}\left(r_{i,t,s}^{U}\right)=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (63)$$

$$\beta_{i,t,s}^{max}\left(R_{i,t}^{D}-r_{i,t,s}^{D}\right)=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (64)$$

$$\beta_{i,t,s}^{min}\left(r_{i,t,s}^{D}\right)=0,\forall i\in I,\forall t\in T,\forall s\in S \quad (65)$$

$$\delta_{i,t}^{max}\left(R_i^{U,max}-R_{i,t}^{U}\right)=0,\forall i\in I,\forall t\in T \quad (66)$$

$$\upsilon_{i,t}^{max}\left(R_i^{D,max}-R_{i,t}^{D}\right)=0,\forall i\in I,\forall t\in T \quad (67)$$

Equations (24)-(33) are consistent with the original upper-level consumer's problem as shown in (1)-(10). Equations (34)-(67) represent the KKT optimality conditions of the lower-level utility's problem (14)-(23). Equations (34)-(42) represents The primal constraints, dual constraints and complementarity slackness conditions are given in equations (34)-(42), (43)-(57) and (58)-(67), respectively.

V. Case Study

To test the proposed bi-level model, we conduct a numerical example of twelve hourly periods (i.e. $t=1,2,\ldots,12$) to schedule the controllable loads with data presented in Table I-VI. Table I-III show the data of type I, II and III loads, respectively. Since an individual appliance is small in terms of demand and consumed energy. We assume that the data in Table I-III are an aggregated value of hundred appliances for meaningful comparison in the example. Table IV lists the data of the generators. For the sake of simplicity, the energy and reserve offers of each generator are consistent throughout the whole scheduled period. We consider three wind power scenarios as shown in Table V. The probabilities of scheduled, high and low wind profiles are 0.6, 0.2 and 0.2 respectively. Table VI shows the hourly demand profile.

TABLE I
TYPE I LOAD DATA

Load j	1
P_j^{I} (MW)	4
H_j^{I}	5

TABLE II
TYPE II LOAD DATA

Load k	1
P_k^{II} (MW)	2.5
p_k^{II}	1
q_k^{II}	2

TABLE II
TYPE III LOAD DATA

Load l	1
$P_l^{III,min}$ (MW)	0
$P_l^{III,max}$ (MW)	5
$E_l^{III,min}$ (MWh)	10

TABLE IV
GENERATOR DATA

Generator i	1	2	3
$P_i^{g,min}$ (MW)	10	10	10
$P_i^{g,max}$ (MW)	30	40	80
$C_{i,t}$ ($/MWh)	20	30	40
$C_{i,t}^{U}$ ($/MWh)	5	4	3.5
$C_{i,t}^{D}$ ($/MWh)	5	4	4.5
$R_i^{U,max}$ (MW)	15	30	10
$R_i^{D,max}$ (MW)	15	30	20

TABLE V
WIND POWER SCENARIOS

Period t	$P_{t,s}^{w}$ (MW) in scenario s		
	$s=1$ (Scheduled)	$s=2$ (High)	$s=3$ (Low)
1	10	20	5
2	15	30	10
3	8	18	3
4	9	13	8
5	18	21	15
6	8	9	6
7	9	10	7
8	12	22	5
9	10	18	9
10	6	9	4
11	5	6	2
12	14	15	11

TABLE VI
BASELOAD POWER

Period t	1	2	3	4	5	6
P_t^{BL}(MW)	90	100	80	100	90	40
t	7	8	9	10	11	12
P_t^{BL}(MW)	100	70	80	100	90	50

Without considering the scheduling of controllable loads, a base case is created to benchmark the optimal results from the proposed model. Given the same set of data and without the consideration of operating period constraints in the consumer's problem, Table VI shows the energy price and the results in the consumer's problem for base and optimal cases. The results under columns "B" and "O" are given by the base case and the optimal load scheduling, respectively.

From Table VI, the average energy price and the total electricity payment for the base case is calculated as $34.6/MWh and $1594/h, respectively. Both of them are more costly than the optimal results which are calculated as $32.6/MWh and $989.8.8/h.

TABLE VI
COMPARISON OF ENERGY PRICE AND DISPATCH RESULTS OF CONSUMER

t	Energy price μ_t ($/MWh)		Type I $x_{1,t}^{I}$		Type II $x_{1,t}^{II}$		Type III $x_{1,t}^{III}$	
	B	O	B	O	B	O	B	O
1	40	35.7	1	0	1	0	1 (5MW)	0
2	38.5	38.5	1	0	0	0	0	0
3	30	31.5	0	1	0	1	0	0
4	40	40	1	0	1	0	1 (5MW)	0
5	30	30	0	1	0	0	0	0
6	40	20	1	1	0	1	0	1 (5MW)
7	40	40	0	0	1	0	0	0
8	27	27	0	0	0	0	0	0
9	30	30	0	1	0	1	0	0
10	40	40	1	0	1	0	0	1
11	40	40	0	0	0	0	0	0 (0.5MW)
12	20	20	0	1	0	1	0	1 (4.5MW)

VI. CONCLUSION

In this paper, we presented a bi-level consumer-utility optimization model with marginal electricity market price to schedule an energy consumption pattern of controllable loads. We classified controllable loads into three types based on their natures and operating characteristics for an upper-level consumer's problem. We further integrated these load models with a stochastic-constrained optimal power flow model for considering stochastic wind power outputs at the lower-level utility's problem. We then converted the bi-level model into a single-level of mathematical program with equilibrium constraints' problem to obtain the optimal load scheduling results. We finally applied the proposed model to a simple case study to demonstrate the feasibility of the method.

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